**Basic Number Theory-2**

* **TUTORIAL**
* [**PROBLEMS**](https://www.hackerearth.com/practice/math/number-theory/basic-number-theory-2/practice-problems/)

The concept of prime numbers is a very important concept in math. This article discusses the concept of prime numbers and related properties.

**What are prime numbers and composite numbers?**

Prime numbers are those numbers that are greater than 1 and have only two factors 1 and itself.

Composite numbers are also numbers that are greater than 1 but they have at least one more divisor other than 1 and itself.

For example, 5 is a prime number because 5 is divisible only by 1 and 5 only. However, 6 is a composite number as 6 is divisible by 1, 2, 3, and 6.

There are various methods to check whether a number is prime.

**Naive approach**

Traverse all the numbers from 11 to NN and count the number of divisors. If the number of divisors is equal to 2 then the given number is prime, else it is not.

void checkprime(int N){

int count = 0;

for( int i = 1;i <= N;++i )

if( N % i == 0 )

count++;

if(count == 2)

cout << N << “ is a prime number.” << endl;

else

cout << N << “ is not a prime number.” << endl;

}

*Time complexity*

The time complexity of this function is *O(N)* because you traverse from 11 to NN.

***Better approach***

If you have two positive numbers NN and DD, such that NN is divisible by DD and DD is less than the square root of NN.

* (N/D)(N/D) must be greater than the square root of NN.
* NN is also divisible by (N/D)(N/D). If there is a divisor of NN that is less than the square root of NN, then there will be a divisor of NN that is greater than square root of NN. You will have to traverse till the square root of NN.

**Note**: You are generating all the divisors of NN and if the count of divisors is greater than 2, then the number is composite.

For example, if N=50, √NN=7 (floor value). You will iterate from 1 to 7 and count the number of divisors of N. The divisors of NN are 1, 50; 2, 25; 5,10. You have 6 divisors of 50, and therefore, it is not prime.

void checkprime(int N) {

int count = 0;

for( int i = 1;i \* i <=N;++i ) {

if( N % i == 0) {

if( i \* i == N )

count++;

else // i < sqrt(N) and (N / i) > sqrt(N)

count += 2;

}

}

if(count == 2)

cout << N << “ is a prime number.” << endl;

else

cout << N << “ is not a prime number.” << endl;

}

*Time complexity*

The time complexity of this function is O(√N)O(N) because you traverse from 1 to √NN.

**Sieve of Eratosthenes**

You can use the *Sieve of Eratosthenes* to find all the prime numbers that are less than or equal to a given number N or to find out whether a number is a prime number.

The basic idea behind the Sieve of Eratosthenes is that at each iteration one prime number is picked up and all its multiples are eliminated. After the elimination process is complete, all the unmarked numbers that remain are prime.

*Pseudo code*

1. Mark all the numbers as prime numbers except 1
2. Traverse over each prime numbers smaller than sqrt(N)
3. For each prime number, mark its multiples as composite numbers
4. Numbers, which are not the multiples of any number, will remain marked as prime number and others will change to composite numbers.

void sieve(int N) {

bool isPrime[N+1];

for(int i = 0; i <= N;++i) {

isPrime[i] = true;

}

isPrime[0] = false;

isPrime[1] = false;

for(int i = 2; i \* i <= N; ++i) {

if(isPrime[i] == true) { //Mark all the multiples of i as composite numbers

for(int j = i \* i; j <= N ;j += i)

isPrime[j] = false;

}

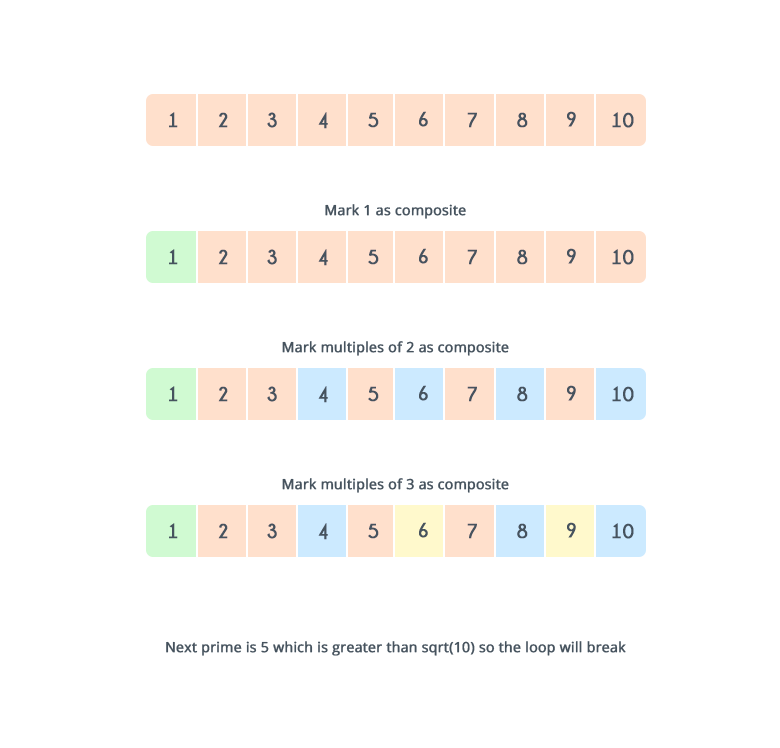
}

}

This code will compute all the prime numbers that are smaller than or equal to N.

Let us compute prime numbers where N=10N=10.

1. Mark all the numbers as prime
2. Mark 1 as composite

In each iteration, check if a number is prime or not, if it is then mark all of its multiple as composite.  


The prime numbers are 2, 3, 5, and 7.

**Time complexity**  
The inner loop that runs for each element is as follows:

1. If i = 2, inner loop runs N / 2 times
2. If i = 3, inner loop runs N / 3 times
3. If i = 5, inner loop runs N / 5 times

*Total complexity*

N \* (½ + ⅓ + ⅕ + … ) = O(NloglogN)

Reference for complexity analysis: [Sieve of Erastothenes](https://en.wikipedia.org/wiki/Sieve_of_Eratosthenes)

**Modification of Sieve of Eratosthenes for fast factorization**

*Factorization in*√NN

vector<int> factorize(int n) {

vector<int> res;

for (int i = 2; i \* i <= n; ++i) {

while (n % i == 0) {

res.push\_back(i);

n /= i;

}

}

if (n != 1) {

res.push\_back(n);

}

return res;

}

For example,

n=50,i=2;res=n=50,i=2;res={2}   
n=25,i=3;res=n=25,i=3;res={2}  
n=25,i=4;res=n=25,i=4;res={2}  
n=25,i=5;res=n=25,i=5;res={2,5,5}  
n=1n=1

When you exit the for loop, resvectorresvector is the factorization of N=50N=50.

At every step, you must look for the prime number of the least value, which divides the current NN. This is the main idea of this modification.

Let us construct an array which will give us this number in **O(1)** time.

int minPrime[n + 1];

for (int i = 2; i \* i <= n; ++i) {

if (minPrime[i] == 0) { //If i is prime

for (int j = i \* i; j <= n; j += i) {

if (minPrime[j] == 0) {

minPrime[j] = i;

}

}

}

}

for (int i = 2; i <= n; ++i) {

if (minPrime[i] == 0) {

minPrime[i] = i;

}

}

Now, use this modification to factorize NN in *O(log(N))* time.

vector<int> factorize(int n) {

vector<int> res;

while (n != 1) {

res.push\_back(minPrime[n]);

n /= minPrime[n];

}

return res;

}

For example,

n=50,minprime[50]=2,res=2n=50,minprime[50]=2,res=2  
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n=5,minprime[5]=5,res=2,5,5n=5,minprime[5]=5,res=2,5,5  
n=1n=1

The required factors are in resres .

**Conditions**

You can implement this modification only if you are allowed to create an array of integers of size NN.

**Note**: This approach is useful when you need to factorize not-very-large numbers. It is not necessary to build a modified Sieve for every problem in which factorization is required. Moreover, you cannot build it for large numbers NN like 109109 or 10121012. Therefore, for such large numbers it is recommended that you factorize in *O(sqrt(N))* instead.

**Fact**

If the factorization of NN is pq11∗pq22∗...∗pqkkp1q1∗p2q2∗...∗pkqk where p1,p2p1,p2...pkpk are the prime factors of NN and q1,q2q1,q2...qkqk are the powers of the respective prime factors, then NN has (q1+1)∗(q2+1)∗...∗(qk+1)(q1+1)∗(q2+1)∗...∗(qk+1) distinct divisors.

**Sieve of Eratosthenes on the segment**:

Sometimes you need to find all the primes that are in the range [L...R][L...R] and not in [1...N][1...N], where RR is a large number.

**Conditions**

You are allowed to create an array of integers with size (R−L+1)(R−L+1).

**Implemention**

bool isPrime[r - l + 1]; //filled by true

for (long long i = 2; i \* i <= r; ++i) {

for (long long j = max(i \* i, (l + (i - 1)) / i \* i); j <= r; j += i) {

isPrime[j - l] = false;

}

}

for (long long i = max(l, 2); i <= r; ++i) {

if (isPrime[i - l]) {

//then i is prime

}

}

The approximate comlexity is *O(sqrt(R))*

**Suggestions**

It is recommended that you do not build a Sieve to check several numbers for primality. Use the following function instead, which works in O(√N)O(N) for every number:

bool isPrime(int n) {

for (int i = 2; i \* i <= n; ++i) {

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*Contributed by:*[*Shubham Gupta*](https://www.hackerearth.com/@shubham1428)

**Did you find this tutorial helpful?**

 YES

 NO

**TEST YOUR UNDERSTANDING**

**Number of Primes**

Given a number N, find number of primes in the range [1,N][1,N].

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The only line of input consists of a number NN.

**Output**:  
Print the number of primes in the range [1,N][1,N].

**Constraints**:  
1≤N≤10000001≤N≤1000000

**SAMPLE INPUT**

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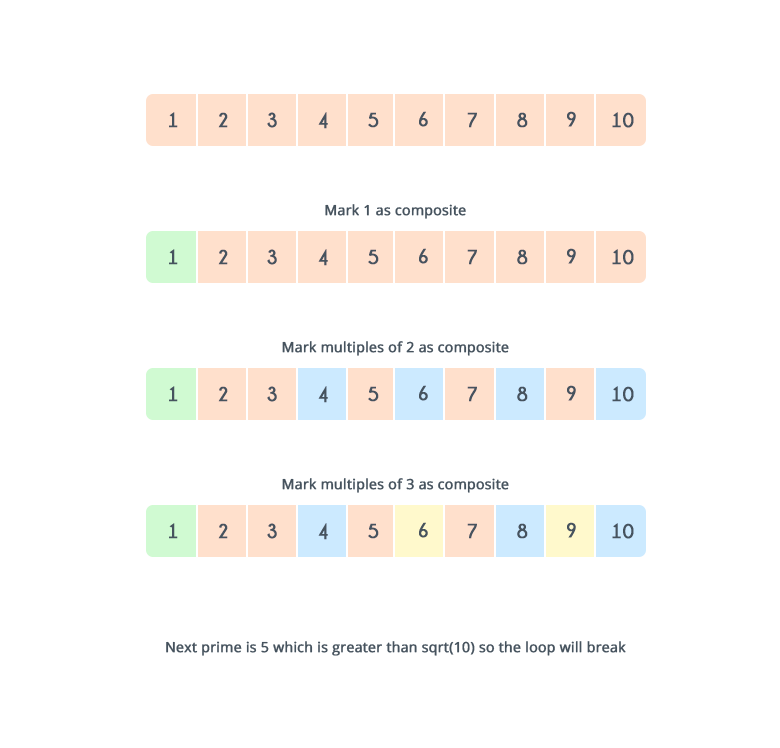
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**SAMPLE OUTPUT**

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<https://www.hackerearth.com/practice/math/number-theory/basic-number-theory-2/tutorial/>

using System;

using System.Collections.Generic;

using System.Linq;

using System.Text;

namespace ConsoleApplication2

{

class Program

{

static int sieve(int N)

{

bool[] isPrime = new bool[N + 1];

for (int i = 0; i <= N; ++i)

{

isPrime[i] = true;

}

isPrime[0] = false;

isPrime[1] = false;

for (int i = 2; i \* i <= N; ++i)

{

if (isPrime[i] == true)

{ //Mark all the multiples of i as composite numbers

for (int j = i \* i; j <= N; j += i)

isPrime[j] = false;

}

}

int cont = 0;

for (int i = 0; i <= N; i++)

{

if (isPrime[i])

{

//Console.Write(i + " ");

cont++;

}

}

return cont;

}

static void Main(string[] args)

{

// sieve(100);

int n = int.Parse(Console.ReadLine());

Console.WriteLine(sieve(n));

Console.ReadLine();

}

}

}